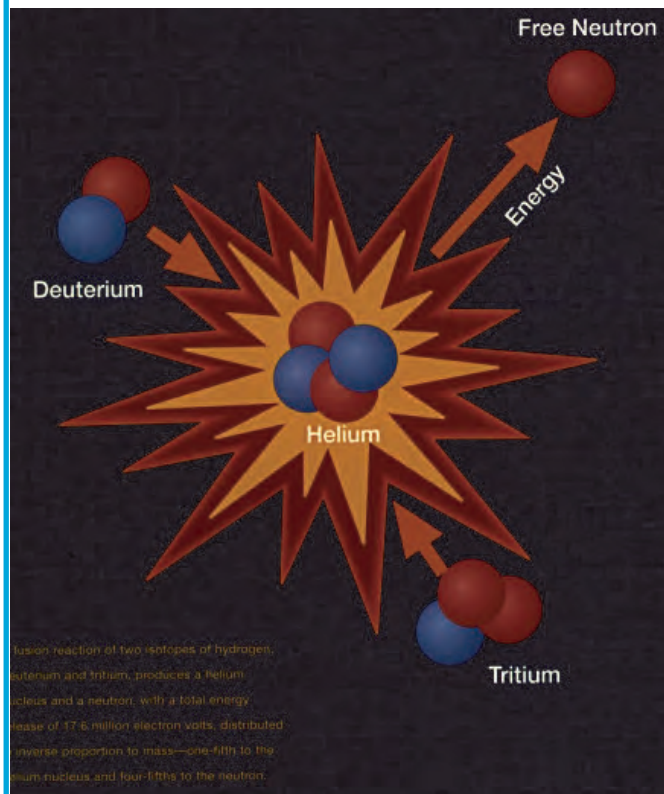


# The fusion parameter space from first principles



**Irvin R. (Irv) Lindemuth**  
**Dept. of Physics, University of Nevada, Reno**  
**formerly (retired, 2003)**  
**Asst. Assoc. Director, Team Ldr., Project Ldr.**  
**Los Alamos National Laboratory**  
**Los Alamos NM**

**Presented at**  
**Fusion Power Associates**  
**Annual Meeting**  
**Washington DC**  
**December 1-2, 2010**

**Acknowledgement: much of this presentation is due to the original insight of Prof. Richard E. Siemon, UNR**

## Abstract

The conventional pathways to fusion—MCF and ICF—have proven to be very long and very expensive. These two approaches are now embodied in two multi-billion-dollar facilities, ITER for MCF and NIF for ICF. These two approaches differ by many orders of magnitude in fundamental physical quantities (density, burn time, fuel pressure, and fuel volume). Given such large differences, it is reasonable to ask an obvious question:

**is there anything in between the extremes of MCF and ICF?**

In this paper, we take a new “first principles” look at the conditions under which fusion can occur. We review the fuel conditions (e.g., confinement time, density, temperature) that must be met to achieve significant fusion energy release. By comparing loss rates with fusion rates, we can identify the density-temperature space where fusion gain can be achieved. This simple analysis offers a general understanding of the extreme differences between the conventional approaches to controlled fusion, MCF and ICF. The analysis shows that the constraint of steady-state operation forces MCF to operate at the low end of the density spectrum and that the constraint of unmagnetized fuel forces ICF to operate at the high end. Most importantly, the analysis shows that using a magnetic field in the fusion fuel allows operation at an intermediate density ( $10^{18}$ - $10^{22}$ /cm<sup>3</sup>), a density range that has many attractive features and potentially overcomes some of the obstacles, particularly cost, faced by the more conventional approaches.

## **FUSION 101--the questions**

- **Under what conditions (fuel density, temperature, magnetic field, etc.) can useful fusion energy release occur? What are the practical limits on these conditions?**
- **Why are there so many orders of magnitude difference in density, volume, power, etc., between NIF (very high density) and ITER (very low density)?**
- **Are three common perceptions correct?  
There are only two viable approaches to fusion--ICF and MCF.  
Fusion is very high cost. Fusion is 30 years away.**
- **Have there been any promising fusion stones left unturned?**
- **Is there anything in between ICF (NIF) and MCF (ITER)?**
- **What would be the cost of a facility to access an intermediate region?**

**“Ballpark” answers--American Journal of Physics Vol. 77, pp. 407-416, May 2009.**

- $\dot{Q}_{loss} = \phi \dot{Q}_{FUS}$ ; find  $n_i, T, B$  so that  $\phi < 1$ ;  $\dot{Q}_{loss} = \dot{Q}_{TC} + \dot{Q}_{RAD}$   
 $\dot{Q}_{RAD} = C_{RAD} n_i^2 T^{1/2}$  (Bremsstrahlung)  $\dot{Q}_{TC} = -\nabla \cdot (K \nabla T)$  ( $K$  = thermal conductivity)

- **Radiation losses determine a minimum temperature:**

$$\frac{\dot{Q}_{FUS}}{\dot{Q}_{RAD}} = \frac{\epsilon_{FUS} n_i^2 \frac{\overline{\sigma V}}{4}}{C_{RAD} n_i^2 T^{1/2}} = \frac{\epsilon_{FUS} \frac{\overline{\sigma V}}{4}}{C_{RAD} T^{1/2}}, \quad \text{independent of } n_i, \quad \frac{\dot{Q}_{FUS}}{\dot{Q}_{RAD}} \geq 1 \quad \text{when } T > 3 \text{ keV}$$

- $\dot{Q}_{TC}, \nabla T$  must be approximated:

$$\dot{Q}_{TC} \approx -\frac{1}{V} \int \nabla \cdot (K \nabla T) dV = -\frac{1}{V} \oint_S K \nabla T \cdot d\vec{S} \approx -\frac{S}{V} K \nabla T \approx \frac{KT}{\gamma \alpha a^2}$$

a = characteristic dimension,  $V = \epsilon a^3$ ,  $\frac{V}{S} = \gamma a$ ,  $\nabla T \approx -\frac{T}{\alpha a}$

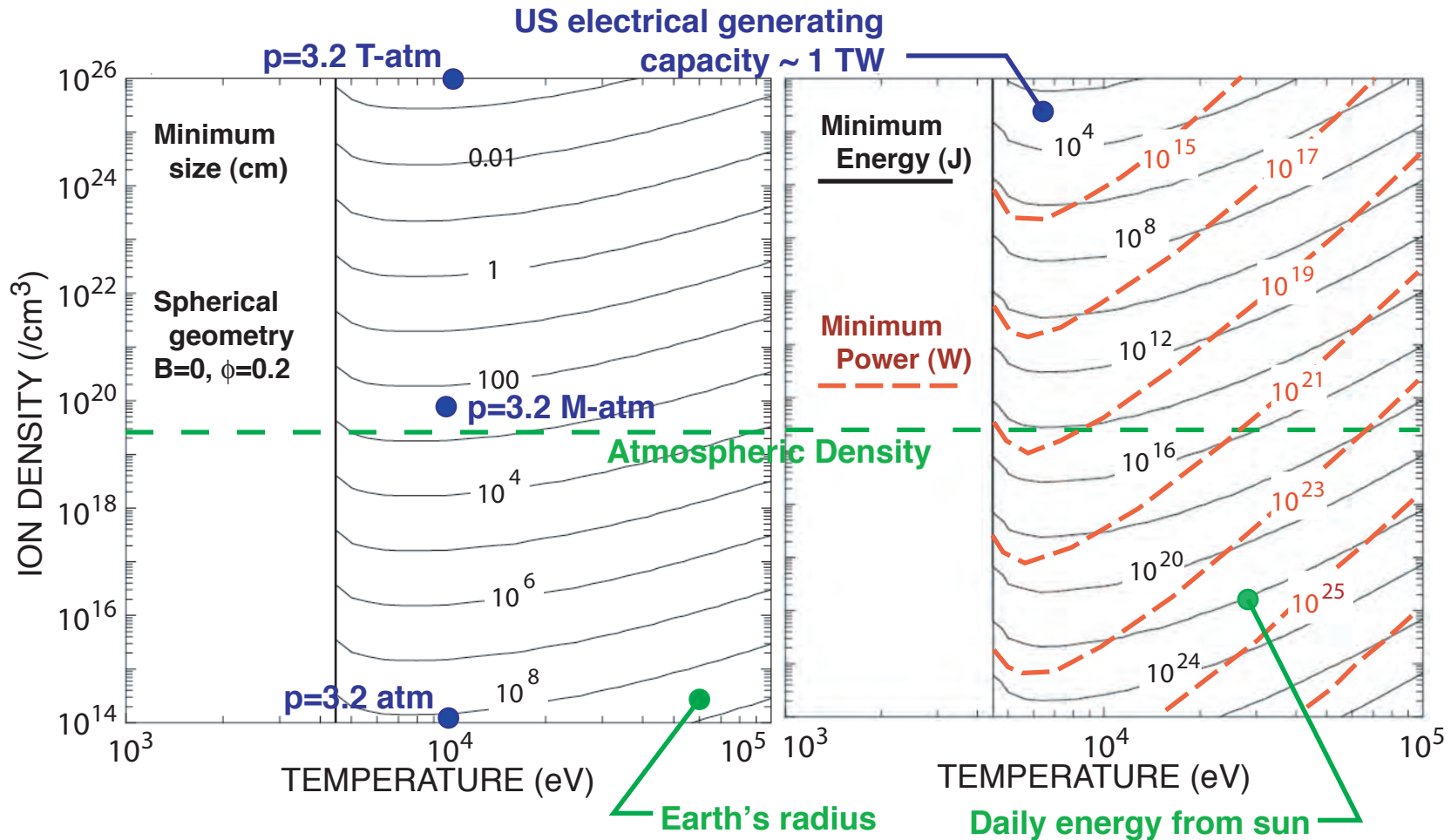
- $\epsilon, \gamma$  are geometric quantities, i.e., for spheres  $\epsilon=4\pi/3, \gamma=1/3$ .
- **Loss rates depend upon  $n_i, T, a$ , model for  $K=K_i+K_e$ , geometry ( $\epsilon, \gamma$ ), profile details ( $\alpha$ ), and, possibly, magnetic field  $B$  (through  $K$ ).**

The conduction rate can be used to determine the minimum system size and other relevant parameters for a desired loss ratio  $\phi$ .

- **Minimum size**  $a_{\min}^2 = \frac{KT}{\gamma\alpha} \frac{1}{\phi\dot{Q}_{FUS} - \dot{Q}_{RAD}}$ ,  $a_{\min} = a_{\min}(n_i, T, B)$
- **Fuel Mass**  $M = n_i(m_i + m_e)\epsilon a_{\min}^3$
- **Fuel thermal energy**  $E_{PLAS} = 3n_i T \epsilon a_{\min}^3$
- **Required heating power**  $P_{HEAT} = (\dot{Q}_{TC} + \dot{Q}_{RAD})\epsilon a_{\min}^3$
- **Required surface heating (intensity)**  $I_{HEAT} = \frac{P_{HEAT}}{S}$
- In the simplest, “classical,” form, the thermal conductivity for an unmagnetized plasma depends only on temperature:  $K_o = C_o T^{5/2}$ .  
With magnetization, the conductivity is reduced by a factor of  $1 + (\omega\tau)^2$ .

# Unmagnetized fuel must operate at very small size, very high density & pressure

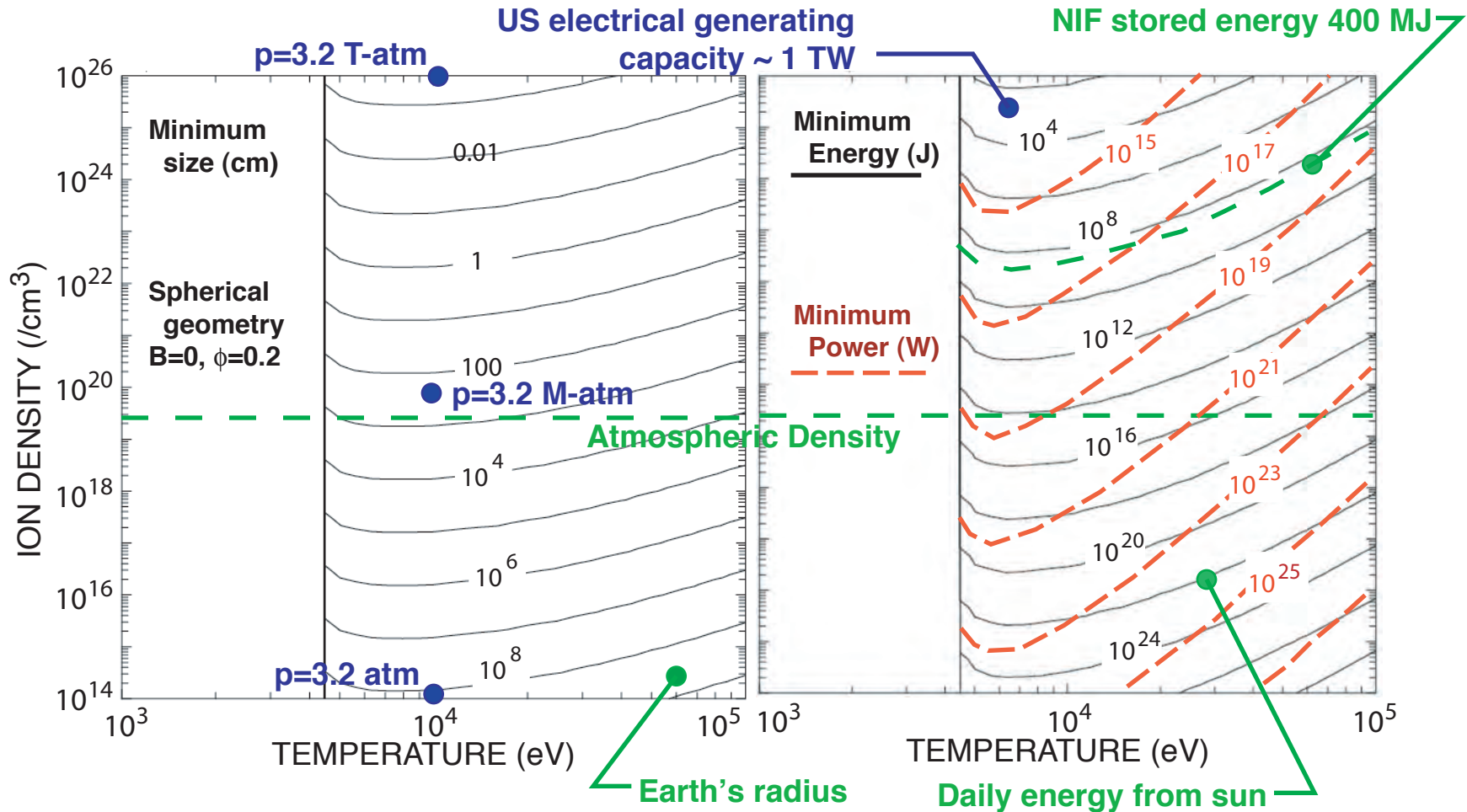
$$P_{atm} = 3.2 \times 10^{-15} n_i T_{keV}$$



- “Steady state” operation requires pressure  $< 1000 \text{ atm.}$ , is not possible, so unmagnetized fuel must be “pulsed,” i.e., a small nuclear explosion.

# Unmagnetized fuel must operate at very small size, very high density & pressure

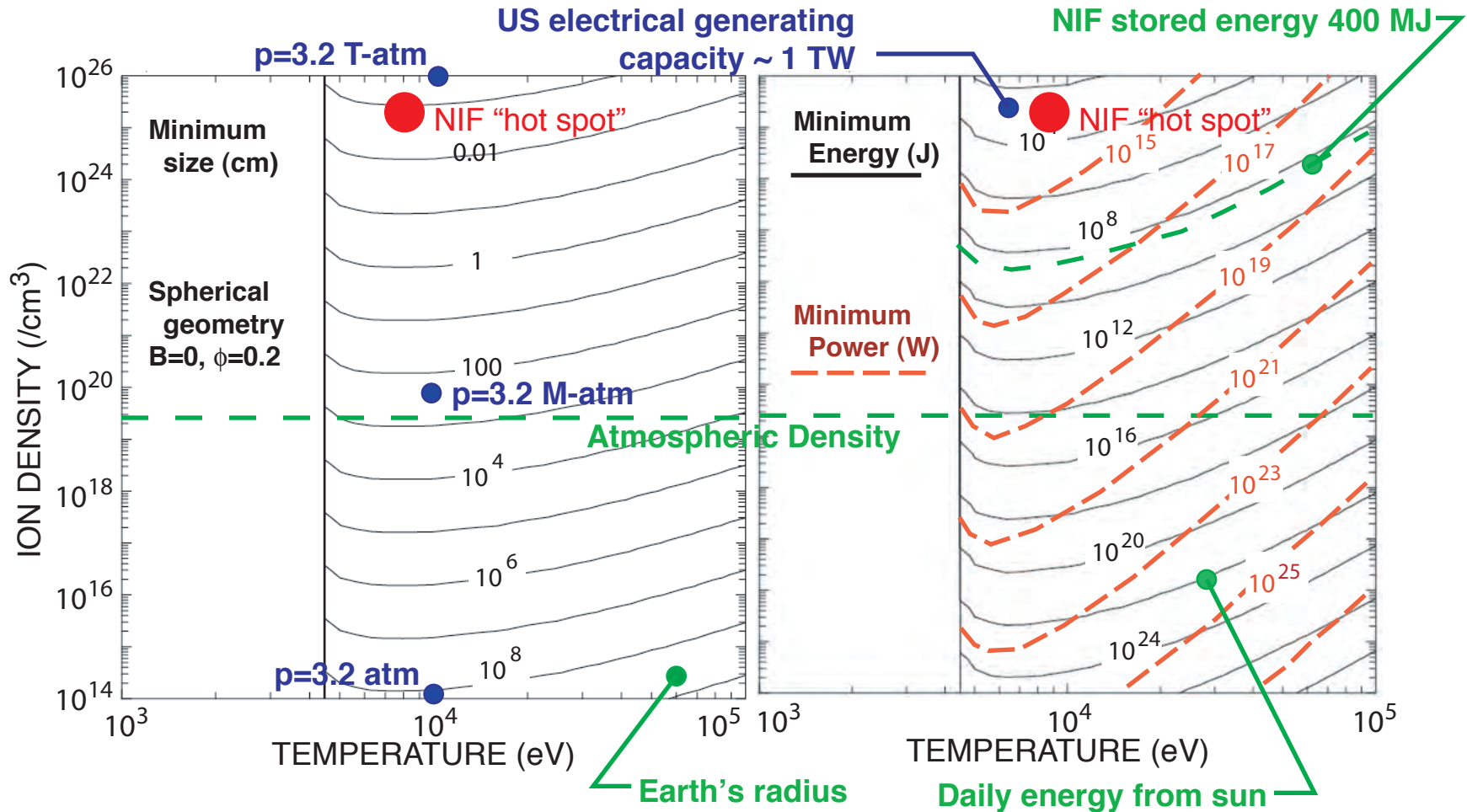
$$P_{atm} = 3.2 \times 10^{-15} n_i T_{keV}$$



- “Steady state” operation requires pressure  $< 1000$  atm., is not possible, so unmagnetized fuel must be “pulsed,” i.e., a small nuclear explosion.

# Unmagnetized fuel must operate at very small size, very high density & very high pressure

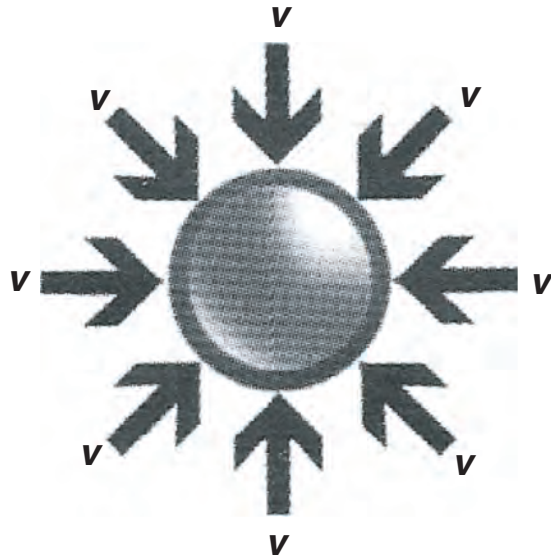
$$P_{atm} = 3.2 \times 10^{-15} n_i T_{keV}$$



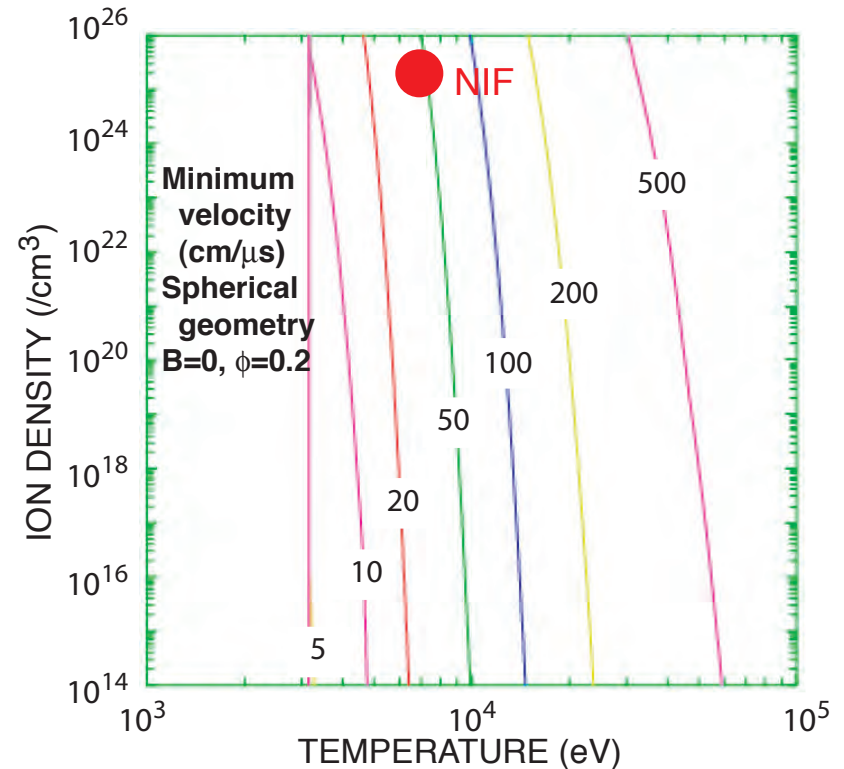
- “Steady state” operation requires pressure  $< 1000 \text{ atm.}$ , is not possible, so unmagnetized fuel must be “pulsed,” i.e., a small nuclear explosion.



## In ICF, the fuel is heated by compressional (hydrodynamic) work of the pusher



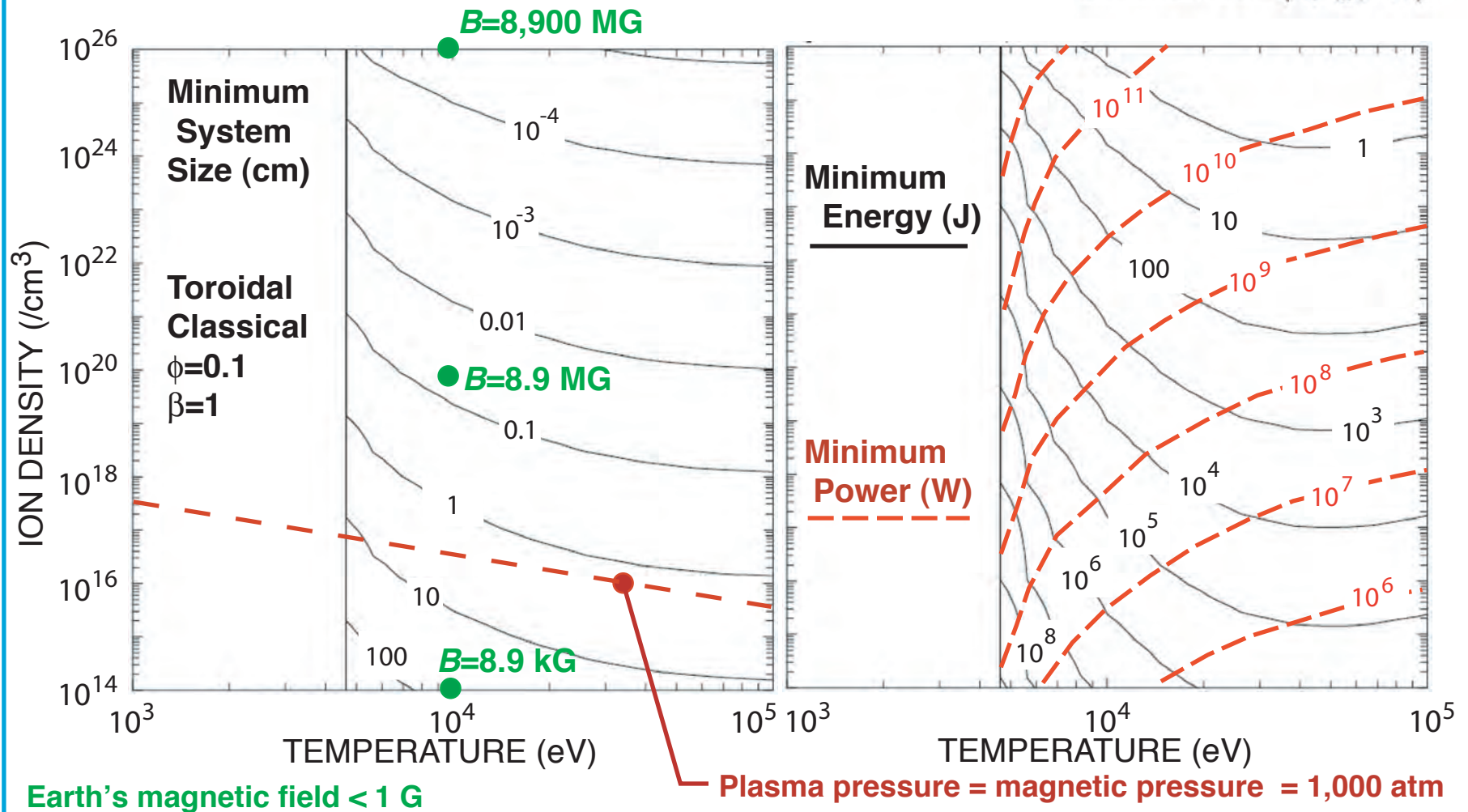
$$V_{\text{IMP}} = \frac{I_{\text{HEAT}}}{p} = \frac{I_{\text{HEAT}}}{2n_i T}$$



- NIF requires an implosion velocity of 40 cm/μs (900,000 mi/hr) and a radial convergence (initial-radius/final-radius) of 30.
- For conventional targets, "the optimal velocity...is the primary determinant of the minimum size driver for ignition..."--J. D. Lindl, UCRL-119015, 11/95.

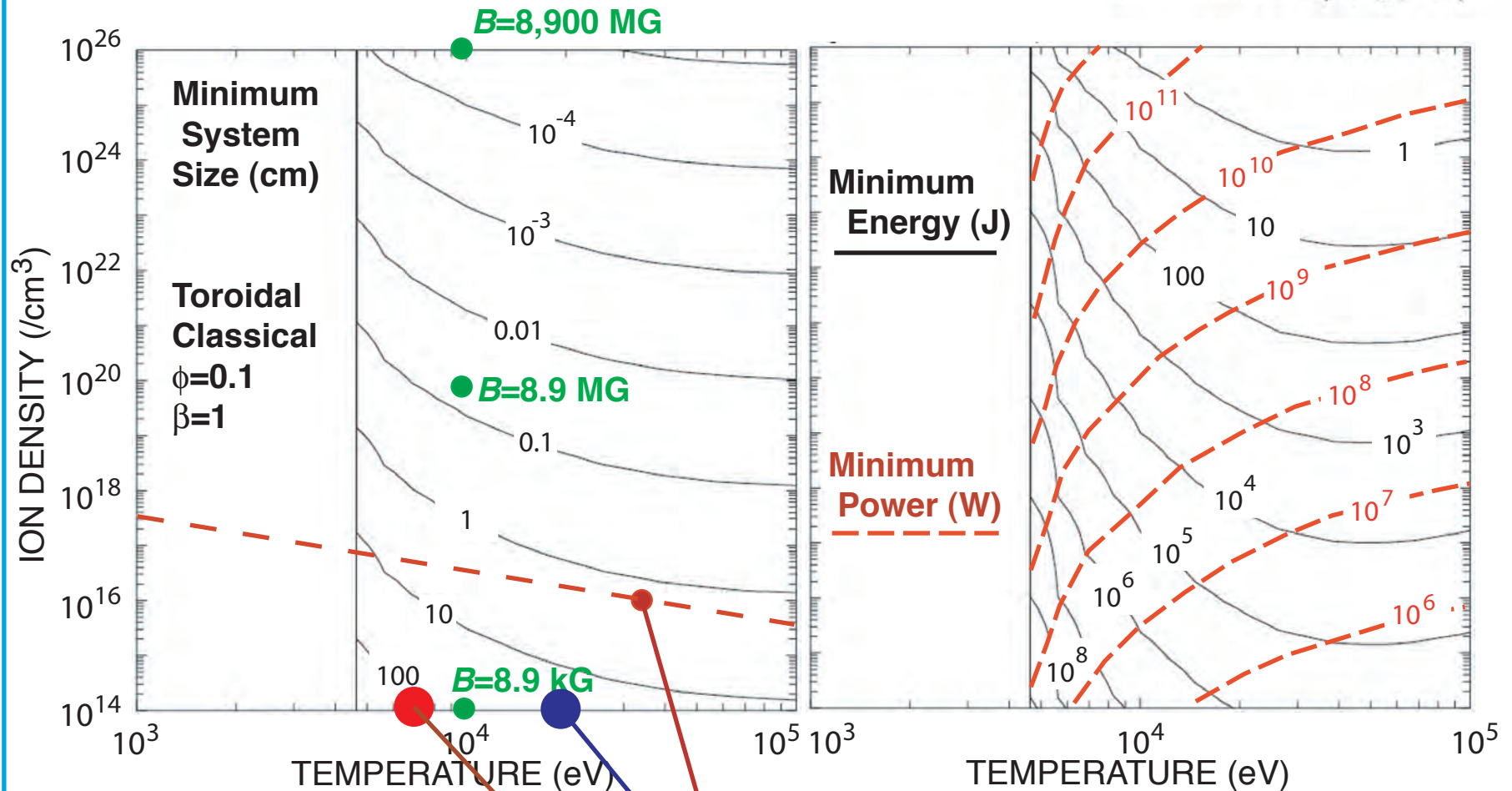
A magnetic field makes “steady state” operation feasible; magnetization reduces the size, energy, and power required.

- Example:  $\beta=1$  ( $\beta$ =plasma pressure/magnetic pressure),  $B = 2.8 \times 10^{-4} (n_i T_{keV} / \beta)^{1/2}$



A magnetic field makes “steady state” operation feasible; magnetization reduces the size, energy, and power required.

- Example:  $\beta=1$  ( $\beta$ =plasma pressure/magnetic pressure),  $B = 2.8 \times 10^{-4} (n_i T_{keV} / \beta)^{1/2}$



Earth's magnetic field < 1 G

Plasma pressure = magnetic pressure = 1,000 atm

TFTR -- 1e14/cm3, 20 keV, 70 cm, 7 MJ, 50 MW

ITER -- 1e14/cm3, 8 keV, 240 cm, 320 MJ, 130 MW

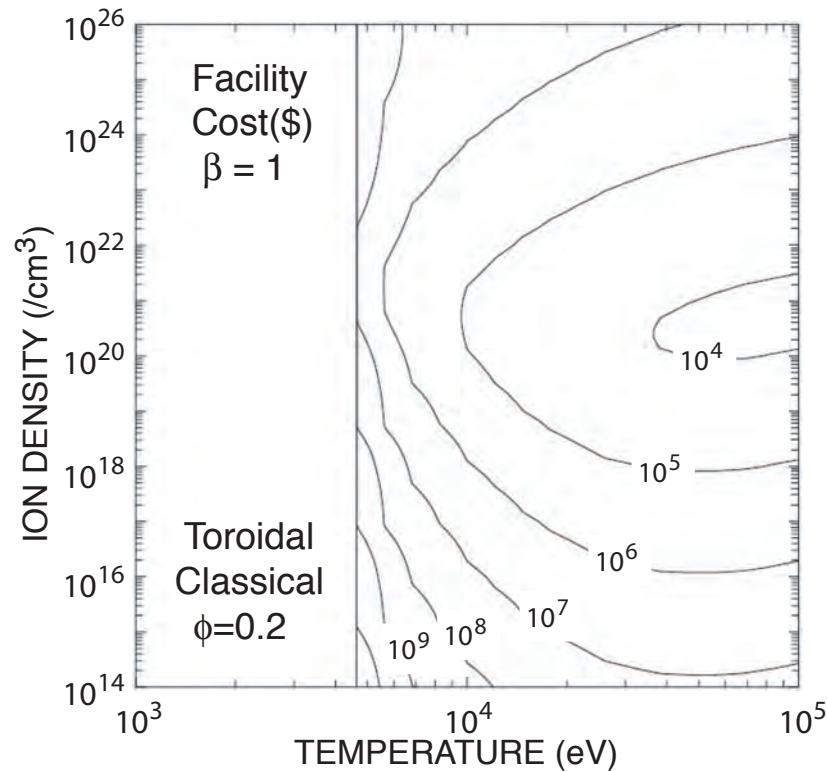
## NIF and ITER differ by factors of 1e4-1e16 in basic physical quantities.

	ITER	Ratio NIF/ITER	NIF
Geometry	Toroidal		Spherical
Cost (\$M)	10,000		3,000
$n_i$ (/cm <sup>3</sup> )	10 <sup>14</sup>	1.4 x 10 <sup>11</sup>	1.4 x 10 <sup>25</sup>
$\rho$ (g/cm <sup>3</sup> )	4.2 x 10 <sup>-10</sup>	1.4 x 10 <sup>11</sup>	57
$T$ (keV)	8		8
$p$ (atm)	2.6	1.4 x 10 <sup>11</sup>	3.6 x 10 <sup>11</sup>
$B$ (kG)	50		0
$\tau_L$ (s)	0.9	1/1.4 x 10 <sup>11</sup>	6.6 x 10 <sup>-12</sup>
$M$ (mg)	350	1/3.5 x 10 <sup>11</sup>	0.01
$a$ (cm)	240	1/6.9 x 10 <sup>4</sup>	3.5 x 10 <sup>-3</sup>
$V$ (m <sup>3</sup> )	8.3 x 10 <sup>2</sup>	1/4.6 x 10 <sup>15</sup>	1.8 x 10 <sup>-13</sup>
$E_{plas}$ (J)	3.2 x 10 <sup>8</sup>	1/3.4 x 10 <sup>4</sup>	9.3 x 10 <sup>3</sup>
$P_{heat}$ (W)	1.3 x 10 <sup>8</sup>	8.5 x 10 <sup>5</sup>	1.1 x 10 <sup>14</sup>
$I_{heat}$ (W/cm <sup>2</sup> )	18	4.2 x 10 <sup>16</sup>	7.5 x 10 <sup>17</sup>

- **Stacks of \$1: 1e4=3.3 ft, 1e12 (bailout)=encircle earth 2.5 times  
1e16=3 round trips to sun**
- **The constraint of unmagnetized fuel forces ICF to operate at high-density, the constraint of “steady-state” forces MCF to operate at low density.  
What if these constraints were relaxed???**

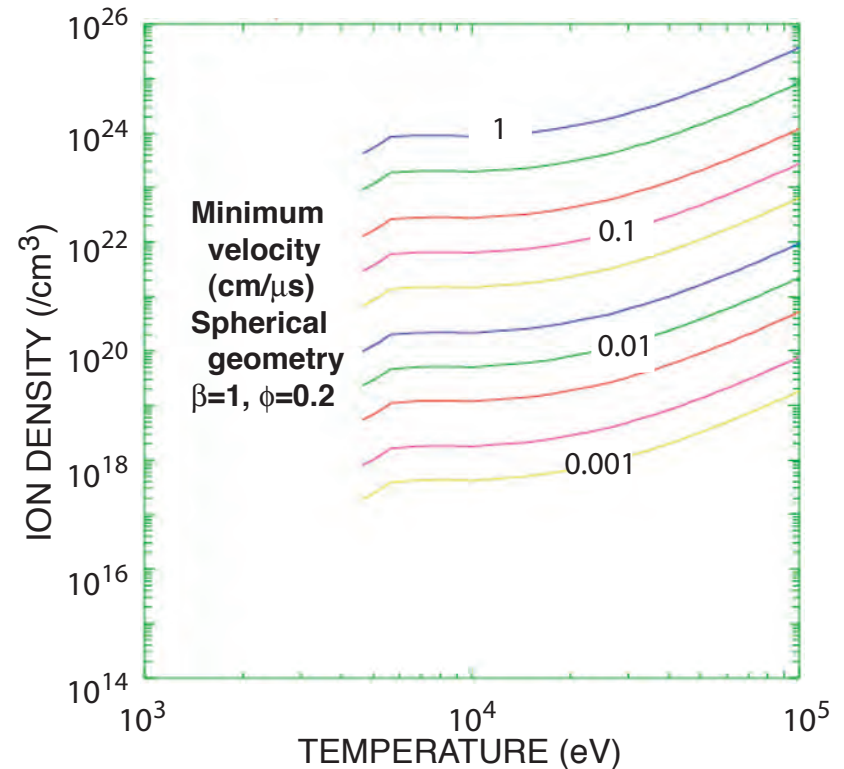
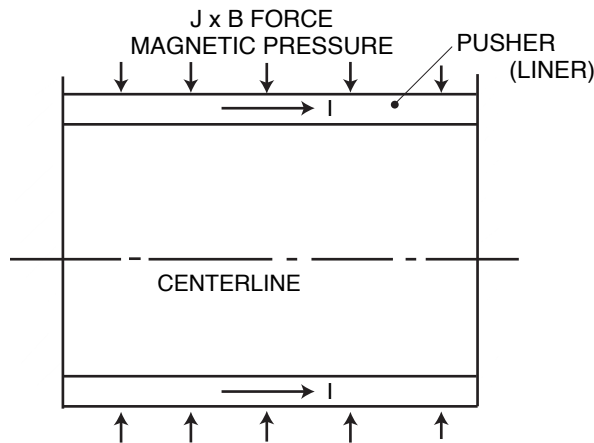
Knowing the cost of ITER and NIF, the cost of fusion facilities in any region of parameter space can be estimated.

$$Cost = c_1 E_{PLAS} + c_2 P_{HEAT} \approx \frac{\$10B}{E_{ITER}} E_{PLAS} + \frac{\$3B}{P_{NIF}} P_{HEAT}$$



- The reduced size/energy (when compared to ITER) and reduced power (when compared to NIF) lead to a very much lower cost at an intermediate density using magnetized fuel.

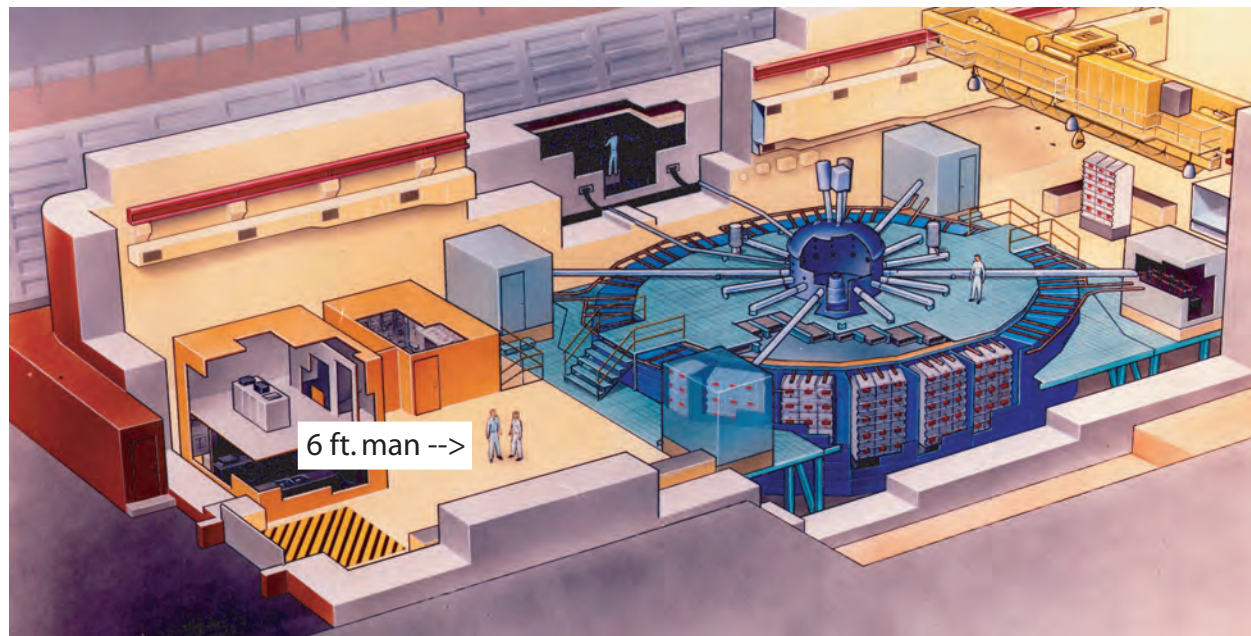
# Can the intermediate density space be accessed? At all? At low cost?



- The velocity required to compress a magnetized plasma by a magnetically driven cylindrical liner is orders of magnitude less than required in ICF.
- An example:  $n_i=10^{20}/\text{cm}^3$ , radius=0.6 cm, length=3.6 cm,  $T=8$  keV,  $B=1$  MG ( $\beta = 65$ )  
Derived:  $M=1.7$  mg,  $\phi = 0.05$ ,  $E_{PLAS}=1.6$  MJ,  $P_{HEAT}=9.0 \times 10^{10}$  W, Cost=\$51M,  $v_{imp}=0.038$  cm/ $\mu$ s
- Assume 10% efficiency, convergence 10 --> need 12-cm-diameter, 3.6-cm-long liner having a kinetic energy of 16 MJ.

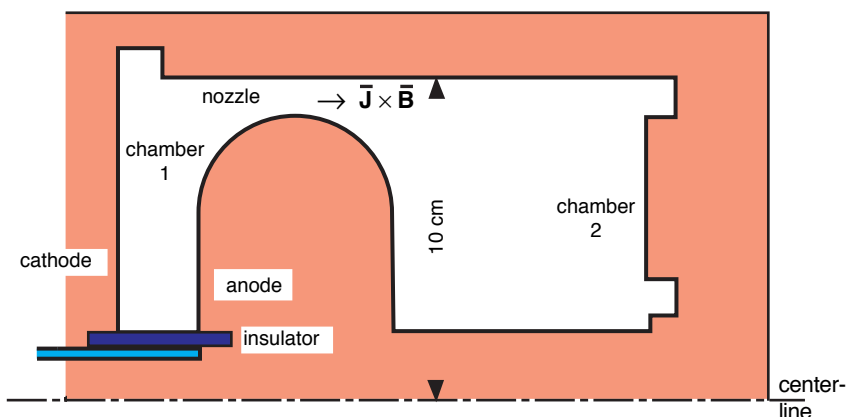
**The Atlas capacitor bank (23 MJ, 30 MA, 6  $\mu$ s) at NTS was designed to drive imploding liners in the range of 1-10 MJ, 0.1-1 cm/ $\mu$ s to create high energy density environments.**

- **Atlas is, serendipitously, an ideal machine for accessing the intermediate density regime by compressing magnetized fuel with a magnetically driven liner.**

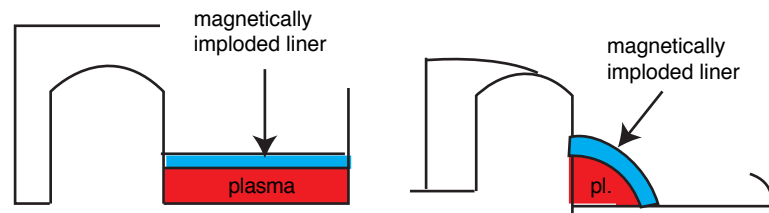


- **Atlas' cost of \$50M confirms the simple cost estimates for fusion facilities.**

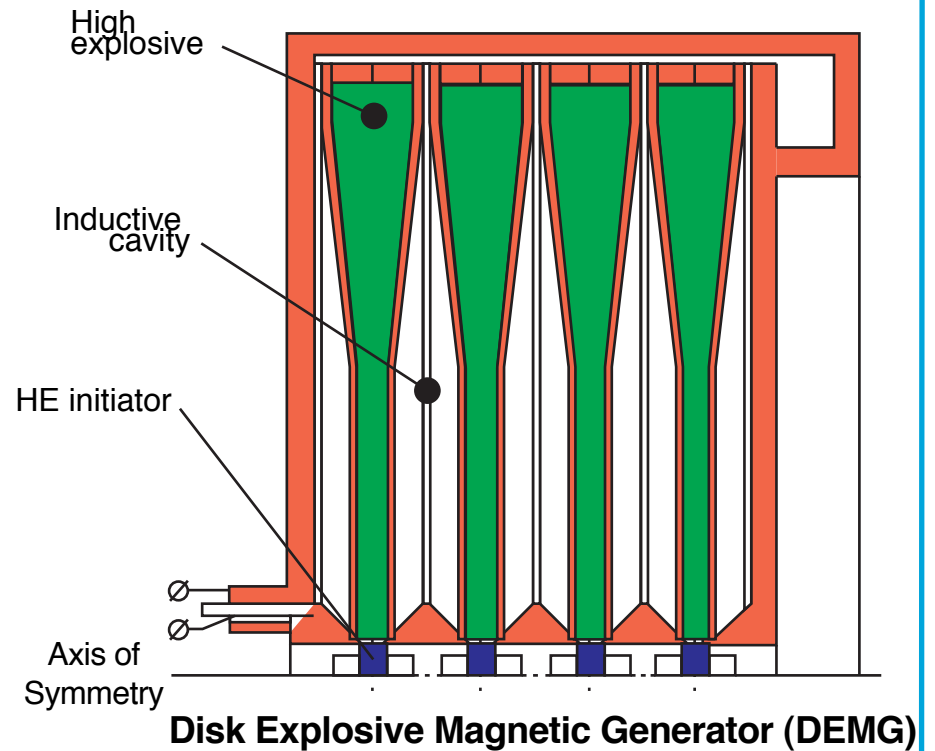
Although other methods can be considered, the Russian “MAGO” plasma formation system creates plasmas having the density and temperature ( $1e18/cm^3$ , 300 eV) required for our example.



**MAGO (Magnitnoye Obzhatiye)**



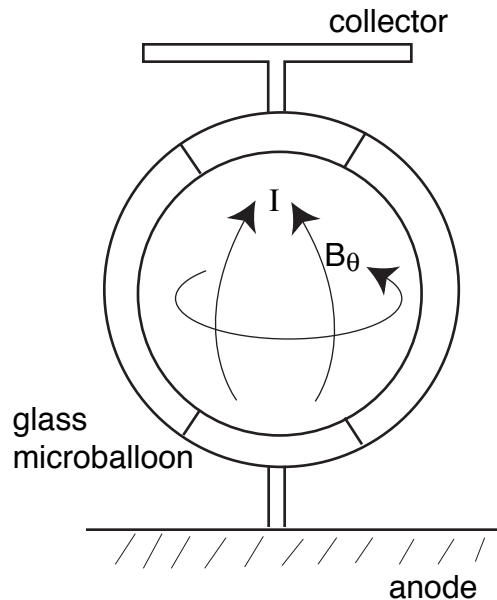
**Implosion scenarios**



- The All-Russian Institute of Experimental Physics (VNIIEF--the “Russian Los Alamos”), building on the work of Nobel Laureate Andre D. Sakharov (“father of Russian H-bomb”), has developed explosively powered generators that develop more electrical current (300 MA) and energy (200 MJ) than any US facility.



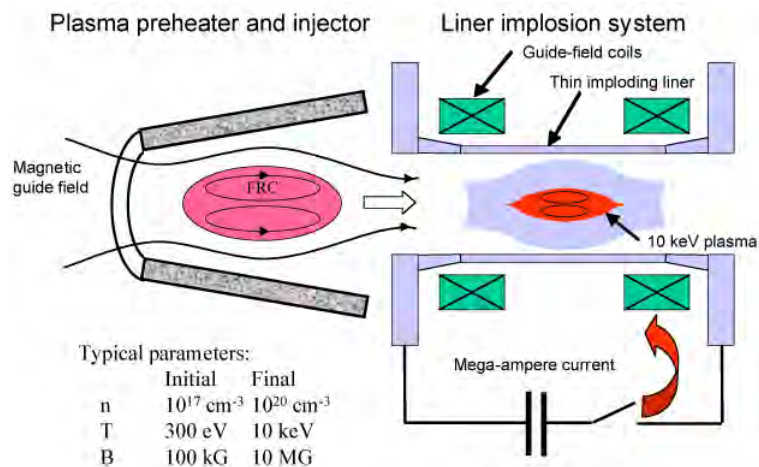
**The first neutrons ever produced by the US particle beam fusion program came from a magnetized target driven by an electron beam (REHYD, 1 MeV, 250 kA, 100 ns, 0.04 TW); see Phys. Today 8/77**



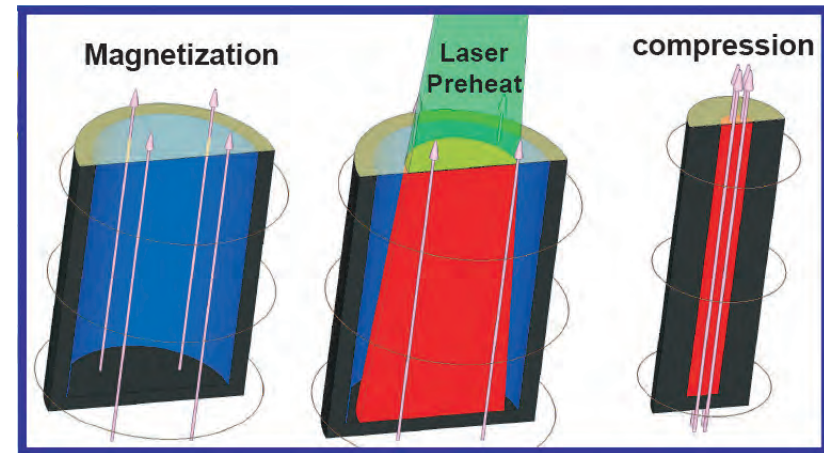
The Sandia " $\Phi$ " Target

- A non-relativistic precursor (5-15 kA, 1  $\mu$ s) was stopped by the collector, creating a voltage which induced an electrical discharge in the fuel.
- The 3-mm-diameter targets imploded at 4 cm/ $\mu$ s.
- $10^6$ - $10^7$  neutrons were observed in CD<sub>2</sub> wire and D-T gas filled ( $6 \times 10^{18}$ /cm<sup>2</sup>) targets.
- No neutrons were observed without the precursor or in a variety of "null" targets.
- Two-dimensional MHD computations indicated a 5-20 eV preheat, 300-500 eV final temperature, consistent with the observed neutron yield (Lindemuth and Widner, Phys. Flu. 24, 1981, p. 746).
- Sandia computations predicted high gain for ion and electron magnetized targets at low intensity (Sweeney and Farnsworth, Nuc. Fus., 1981, p. 41).

## Many combinations of plasma formation and implosion drivers appear possible for accessing the intermediate density regime



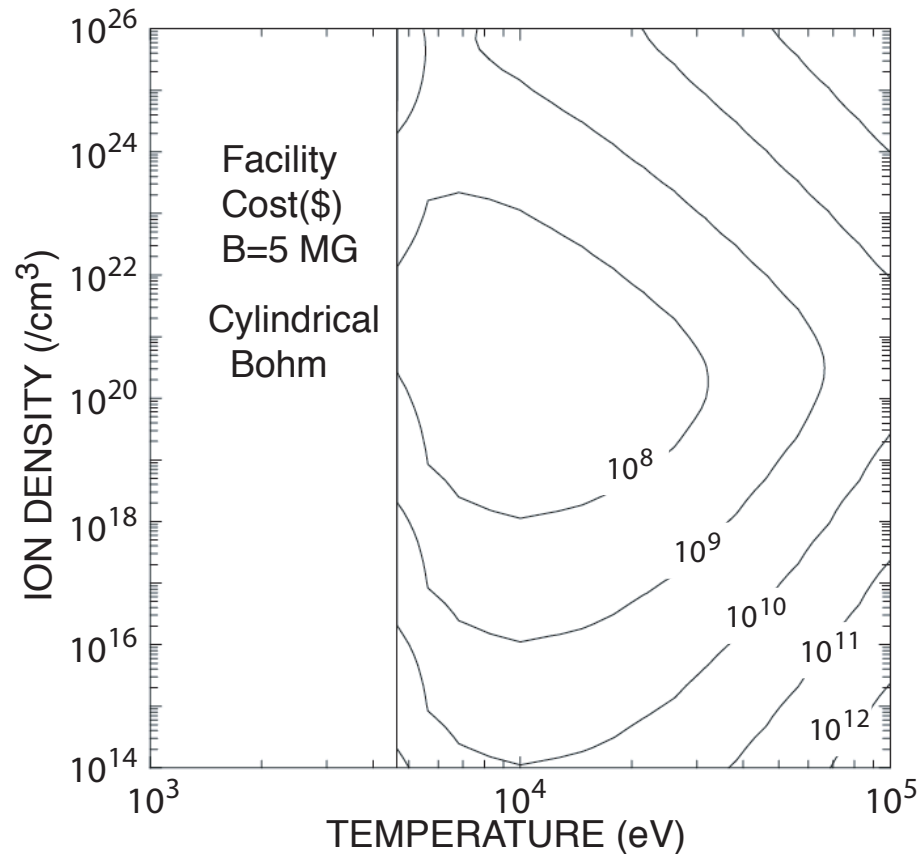
AFRL/LANL/UNR FRC/Shiva-Star  
(J. Degnan, G. Wurden et al.)



SNL "Z" MAGLIF MAGnetized Liner Inertial Fusion  
(S. Slutz et al.)

- **Compression of a magnetized plasma by an imploding pusher/liner was named Magnetized Target Fusion (MTF) by LANL in 1991; more recently, the term Magneto-Inertial Fusion (MIF) has been used.**
- **MTF/MIF does not have the severe density constraints of MCF and ICF. MTF/MIF may be possible over a density range covering 4-6 orders of magnitude.**

## But, what if the thermal losses are Bohm-like rather than classical?



- Computations by Dawson and experiments at Columbia U. suggest that the losses should be classical, but even if the losses are Bohm, there is a large intermediate space where MTF should be lower cost than ICF, MCF

## Controlled Fusion is a long-term, expensive proposition--or is it????

	<i>ITER</i>	<i>MTF</i> <i>example</i>	<i>NIF</i>
<i>Cost (\$M)</i>	10,000	51	3,000
$n_i$ (/cm <sup>3</sup> )	10 <sup>14</sup>	10 <sup>20</sup>	1.4 x 10 <sup>25</sup>
$\rho$ (g/cm <sup>3</sup> )	4.2 x 10 <sup>-10</sup>	4.2 x 10 <sup>-4</sup>	57
<i>T (keV)</i>	8	8	8
<i>p (atm)</i>	2.6	2.6 x 10 <sup>6</sup>	3.6 x 10 <sup>11</sup>
<i>B (kG)</i>	50	1,000	0
$\tau_L$ (s)	0.9	9 x 10 <sup>-7</sup>	6.6 x 10 <sup>-12</sup>
<i>M (mg)</i>	350	1.7	0.01
<i>a (cm)</i>	240	0.6	3.5 x 10 <sup>-3</sup>
<i>V (m<sup>3</sup>)</i>	8.3 x 10 <sup>2</sup>	4.0 x 10 <sup>-6</sup>	1.8 x 10 <sup>-13</sup>
<i>E<sub>plas</sub> (J)</i>	3.2 x 10 <sup>8</sup>	1.6 x 10 <sup>6</sup>	9.3 x 10 <sup>3</sup>
<i>P<sub>heat</sub> (W)</i>	1.3 x 10 <sup>8</sup>	9.0 x 10 <sup>10</sup>	1.1 x 10 <sup>14</sup>
<i>I<sub>heat</sub> (W/cm<sup>2</sup>)</i>	18	1.0 x 10 <sup>10</sup>	7.5 x 10 <sup>17</sup>

- ICF and MCF differ by 10<sup>10</sup>--10<sup>12</sup> in fuel density and time scale and by more than 10<sup>15</sup> in burning fuel volume. **The vast parameter space between these two extremes is unexplored.**
- MTF can be investigated using machines that already exist (e.g., Atlas \$50M).
- The low cost and size of experimental facilities should significantly reduce fusion's development time.
- Unfortunately, unless the US program adopts a "balanced portfolio" approach, MTF (and other alternate concepts) will never have a chance to reach technical maturity.